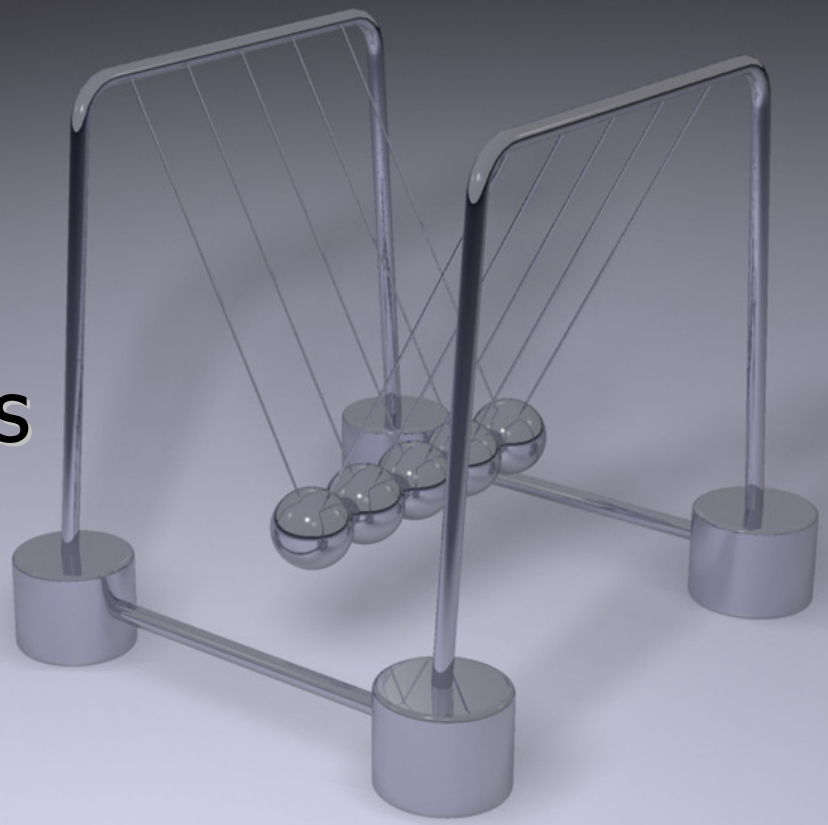
DM2212
Programming Physics



FALLING APPLES
MINDPHASE

ML: Dioselin Gonzalez

2007 S1

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Vector theory review

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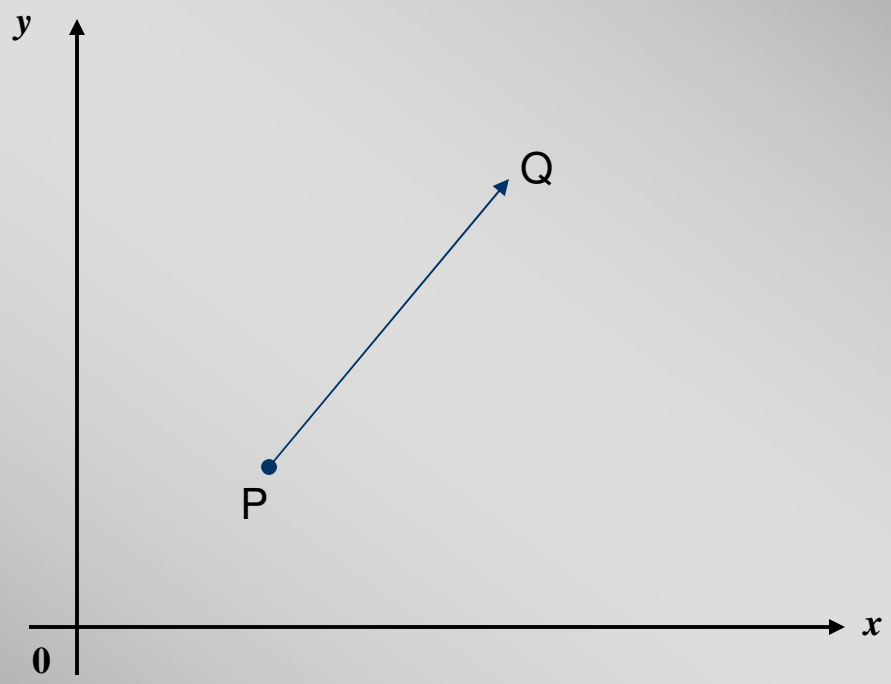
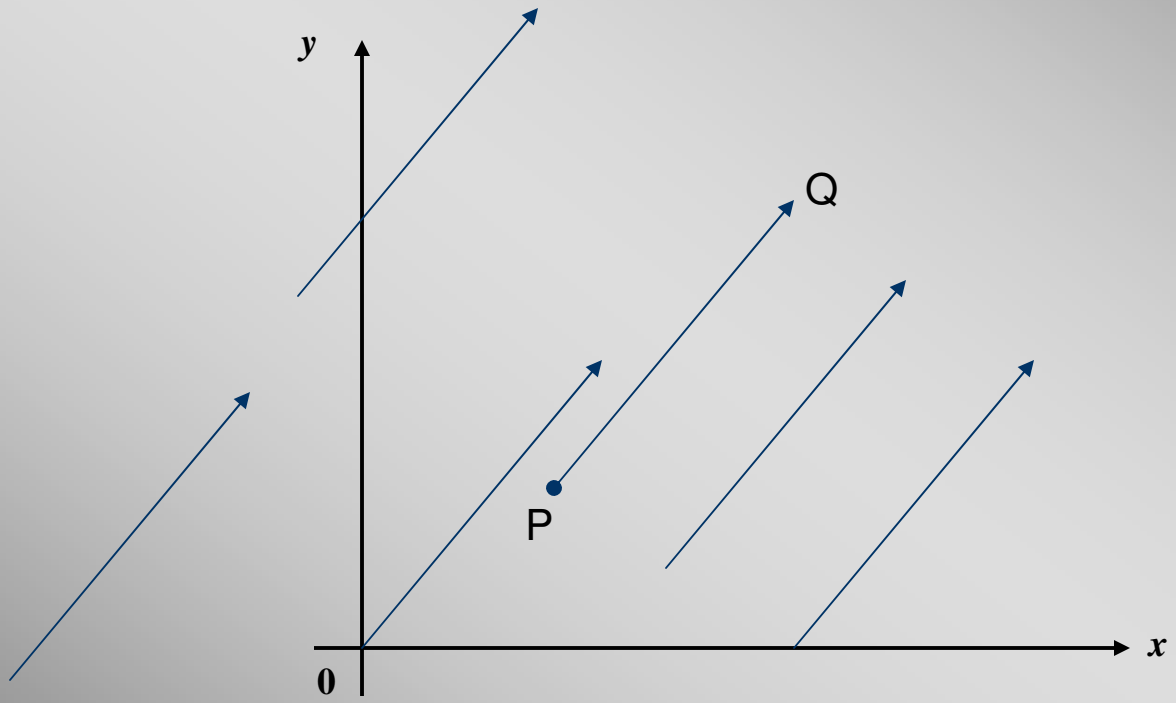
Directed line segment



$$\overrightarrow{PQ} \neq \overrightarrow{QP}$$

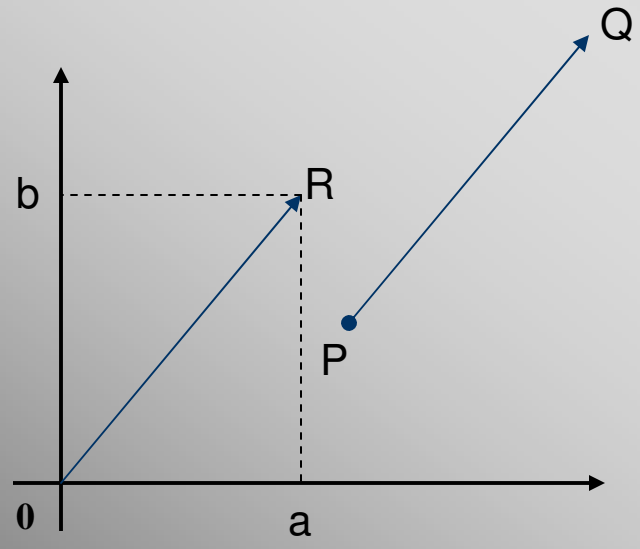
Equivalent directed line segments



Same magnitude and direction

Vector

- Geometric definition:
  - The set of all directed line segments that are equivalent to a given directed segment. Any segment from that set is known as *representative* of the vector.



$\mathbf{v} = (a,b)$

Vector

- Algebraic definition:
  - A vector **v** in $\mathbb{R}^3$ is an ordered triad of real numbers *(x,y,z)*
  - *x*, *y* and *z* are known as the *components* of vector **v**
  - The *zero vector* is (0,0,0)

Both definitions describe the exact same objects!

Vector's magnitude

- Magnitude of $\mathbf{v} = (x,y,z)$

  $|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$

- Unit vector

  $|\mathbf{v}| = 1$

  What's the distance between $P=(x_1,y_1,z_1)$ and $Q=(x_2,y_2,z_2)$?

Vector operations

Let $\mathbf{u}=(x_1,y_1,z_1)$ and $\mathbf{v}=(x_2,y_2,z_2)$ be two vectors and α a real number (scalar)

$$\mathbf{u}+\mathbf{v} = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$$

$$\alpha\,\mathbf{u} = (\alpha\, x_1,\ \alpha\, y_1,\ \alpha\, z_1)$$

Vector's direction

- The direction of vector **v** in $\mathbb{R}^3$ is defined as the direction of unit vector **u**=**v**/|**v**|

$$\cos\alpha = \frac{x}{|\mathbf{v}|} \qquad \cos\beta = \frac{y}{|\mathbf{v}|} \qquad \cos\gamma = \frac{z}{|\mathbf{v}|}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = ?$$

Dot product

- Let $\mathbf{u}=(x_1,y_1,z_1)$ and $\mathbf{v}=(x_2,y_2,z_2)$ be two vectors

$$\mathbf{u} \bullet \mathbf{v} = x_1x_2 + y_1y_2 + z_1 z_2$$

$$\mathbf{u} \bullet \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}|\ \cos\varphi$$

u and **v** are parallel if φ is cero or π, and orthogonal (or perpendicular) if ?

http://www.ies.co.jp/math/java/trig/graphCosX/graphCosX.html

Cross product

- Let $\mathbf{u}=(a_1,b_1,c_1)$ and $\mathbf{v}=(a_2,b_2,c_2)$ be two vectors

$$\mathbf{u} \times \mathbf{v} = (b_1c_2-c_1b_2,\ c_1a_2-a_1c_2,\ a_1b_2-b_1a_2\ )$$

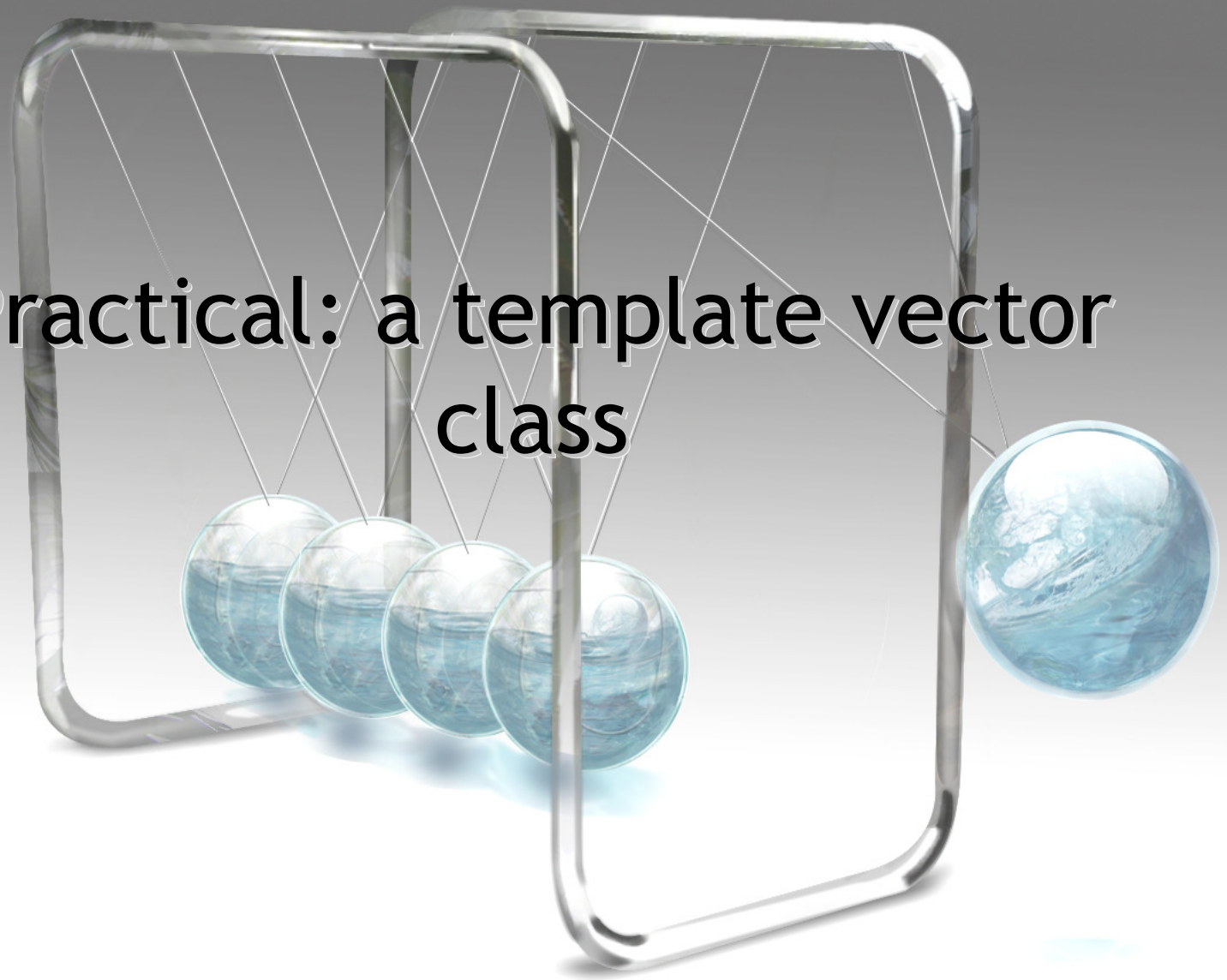
Easier way: determinant notation

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \mathbf{i}\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Cross product theorems

1. $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
2. $(\mathbf{u} \times \mathbf{v}) = -(\mathbf{v} \times \mathbf{u})$
3. $(\alpha\,\mathbf{u}) \times \mathbf{v} = \alpha\,(\mathbf{u} \times \mathbf{v})$
4. $\mathbf{u} \times (\mathbf{v}+\mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
5. $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w} = \mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})$
6. $\mathbf{u} \bullet (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \bullet (\mathbf{u} \times \mathbf{v}) = 0$
7. If $\mathbf{u}$ and $\mathbf{v}$ are parallel then $\mathbf{u} \times \mathbf{v} = \mathbf{0}$
8. $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\ |\mathbf{v}| \sin \varphi$

Practical: a template vector class



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C++ templates

```
template <class identifier> function_declaration;
template <typename identifier> function_declaration;
template <class identifier> class_declaration;
template <typename identifier> class_declaration;
```

```
template <class T>
T getMax (T a, T b){
   return ((a>b) ? a : b);
}
```

```
void main(){
   int a,b,c;
   a = 10; b=15;
   c = getMax<int>(a,b);
   //also: c = getMax(a,b);
}
```

C++ templates

```
template <class T, int N>
class mysequence {
    T memblock [N];
  public:
    void setmember (int x, T value);
    T getmember (int x);
};

void main(){
  mysequence<char, 5>;
  mysequence.setmember(1,'h');
  mysequence.setmember(2,'i');
  mysequence.setmember(1,0);

}
```

Today's lab: template vector3 class

- Methods:
  - Copy constructor
  - Magnitude
  - Normalize
  - Direction
- Operators and functions:
  - * and / (multiplication and division by scalar)
  - * (dot product)
  - + , -, =
  - Cross product
  - Angle between two vectors

References

- http://www.cplusplus.com/doc/tutorial/templates.html
- http://www.devarticles.com/c/a/Cplusplus/The-Mighty-Cplus-Template/